The Riemann Zeta Function Theory And Applications Aleksandar Ivic

Zeta and L-Functions of Varieties and Motives

Theory of Functions

Lectures on the Riemann Zeta-function

Automorphic Forms, Representation Theory and Arithmetic

The subject of this book is probabilistic number theory. In a wide sense probabilistic number theory is part of the analytic number theory, where the methods and ideas of probability theory are used to study the distribution of values of arithmetic objects. This is usually complicated, as it is difficult to say anything about their concrete values. This is why the following problem is usually investigated: given some set, how often do values of an arithmetic object get into this set? It turns out that this frequency follows strict mathematical laws. Here we discover an analogy with quantum mechanics where it is impossible to describe the chaotic behaviour of one particle, but that large numbers of particles obey statistical laws. The objects of investigation of this book are Dirichlet series, and, as the title shows, the main attention is devoted to the Riemann zeta-function. In studying the distribution of values of Dirichlet series the weak convergence of probability measures on different spaces (one of the principal asymptotic probability theory methods) is used. The application of this method was launched by H. Bohr in the third decade of this century and it was implemented in his works together with B. Jessen. Further development of this idea was made in the papers of B. Jessen and A. Wintner, V. Borchsenius and B.

Graph theory meets number theory in this stimulating book. Ihara zeta functions of finite graphs are reciprocals of polynomials, sometimes in several variables. Analogies abound with number-theoretic functions such as Riemann/Dedekind zeta functions. For example, there is a Riemann hypothesis (which may be false) and prime number theorem for graphs. Explicit constructions of graph coverings use Galois theory to generalize Cayley and Schreier graphs. Then non-isomorphic simple graphs with the same zeta are produced, showing you cannot hear the shape of a graph. The spectra of matrices such as the adjacency and edge adjacency matrices of a graph are essential to the plot of this book, which makes connections with quantum chaos and random matrix theory, plus expander/Ramanujan graphs of interest in computer science. Created for beginning graduate students, the book will also appeal to researchers. Many well-chosen illustrations and exercises, both theoretical and computer-based, are included throughout.

Studying the relationship between the geometry, arithmetic and spectra of fractals has been a subject of significant interest in contemporary mathematics. This book contributes to the literature on the subject in several different and new ways. In particular, the authors provide a rigorous and detailed study of the spectral operator, a map that sends the geometry of fractal
strings onto their spectrum. To that effect, they use and develop methods from fractal geometry, functional analysis, complex analysis, operator theory, partial differential equations, analytic number theory and mathematical physics. Originally, M L Lapidus and M van Frankhuysen 'heuristically' introduced the spectral operator in their development of the theory of fractal strings and their complex dimensions, specifically in their reinterpretation of the earlier work of M L Lapidus and H Maier on inverse spectral problems for fractal strings and the Riemann hypothesis. One of the main themes of the book is to provide a rigorous framework within which the corresponding question 'Can one hear the shape of a fractal string?' or, equivalently, 'Can one obtain information about the geometry of a fractal string, given its spectrum?' can be further reformulated in terms of the invertibility or the quasi-invertibility of the spectral operator. The infinitesimal shift of the real line is first precisely defined as a differentiation operator on a family of suitably weighted Hilbert spaces of functions on the real line and indexed by a dimensional parameter c. Then, the spectral operator is defined via the functional calculus as a function of the infinitesimal shift. In this manner, it is viewed as a natural 'quantum' analog of the Riemann zeta function. More precisely, within this framework, the spectral operator is defined as the composite map of the Riemann zeta function with the infinitesimal shift, viewed as an unbounded normal operator acting on the above Hilbert space. It is shown that the quasi-invertibility of the spectral operator is intimately connected to the existence of critical zeros of the Riemann zeta function, leading to a new spectral and operator-theoretic reformulation of the Riemann hypothesis. Accordingly, the spectral operator is quasi-invertible for all values of the dimensional parameter c in the critical interval (0,1) (other than in the midfractal case when c = 1/2) if and only if the Riemann hypothesis (RH) is true. A related, but seemingly quite different, reformulation of RH, due to the second author and referred to as an 'asymmetric criterion for RH', is also discussed in some detail: namely, the spectral operator is invertible for all values of c in the left-critical interval (0,1/2) if and only if RH is true. These spectral reformulations of RH also led to the discovery of several 'mathematical phase transitions' in this context, for the shape of the spectrum, the invertibility, the boundedness or the unboundedness of the spectral operator, and occurring either in the midfractal case or in the most fractal case when the underlying fractal dimension is equal to 1/2 or 1, respectively. In particular, the midfractal dimension c = 1/2 is playing the role of a critical parameter in quantum statistical physics and the theory of phase transitions and critical phenomena. Furthermore, the authors provide a 'quantum analog' of Voronin's classical theorem about the universality of the Riemann zeta function. Moreover, they obtain and study quantized counterparts of the Dirichlet series and of the Euler product for the Riemann zeta function, which are shown to converge (in a suitable sense) even inside the critical strip. For pedagogical reasons, most of the book is devoted to the study of the quantized Riemann zeta function. However, the results obtained in this monograph are expected to lead to a quantization of most classic arithmetic zeta functions, hence, further 'naturally quantizing' various aspects of analytic number theory and arithmetic geometry. The book should be accessible to experts and non-experts alike, including mathematics and physics graduate students and postdoctoral researchers, interested in fractal geometry, number theory, operator theory and functional analysis, differential equations, complex analysis, spectral theory, as well as mathematical and theoretical physics. Whenever necessary, suitable background about the different subjects involved is provided and the new work is placed in its proper historical context. Several appendices supplementing the main text are also included.

**The Theory of the Riemann Zeta-function with Application**

The Riemann zeta function was introduced by L. Euler (1737) in connection with questions about the distribution of prime numbers. Later, B. Riemann (1859) derived deeper results about the prime numbers by considering the zeta function in the complex variable. The famous Riemann Hypothesis, asserting that all of the non-trivial zeros of zeta are on a critical line in the complex plane, is one of the most important unsolved problems in modern mathematics. The present book consists of two parts. The first part covers classical material about the zeros of the Riemann zeta function with applications to the distribution of prime numbers, including those made by Riemann himself, F. Carlson, and Hardy-Littlewood. The second part gives a complete presentation of Levinson's method for zeros on the critical line, which allows one to prove, in particular, that more than one-third of non-trivial zeros of zeta are on the critical line. This approach and some results concerning integrals of Dirichlet polynomials are new. There are also technical lemmas which can be useful in a broader context.

**The Riemann Zeta-function**

The main topic of this book is the deep relation between the spacings between zeros of zeta and $L$-functions and spacings between eigenvalues of random elements of large compact classical groups. The authors draw upon many disparate areas of mathematics from algebraic geometry, moduli spaces, monodromy, equidistribution, and the Weil conjectures to probability theory and the compact classical groups.

**The Riemann Zeta-function: the Theory of the Riemann Zeta-function with Applications**

Monograph on most important topic in number theory.

**The Riemann Zeta-Function**
The Bloch–Kato Conjecture for the Riemann Zeta Function

Originally published in 1934, this volume presents the theory of the distribution of the prime numbers in the series of natural numbers. Despite being long out of print, it remains unsurpassed as an introduction to the field.

An Introduction to the Theory of the Riemann Zeta-Function

Prime Numbers and the Riemann Hypothesis

Riemann’s Zeta Function

The amount of mathematics invented for number-theoretic reasons is impressive. It includes much of complex analysis, the re-foundation of algebraic geometry on commutative algebra, group cohomology, homological algebra, and the theory of motives. Zeta and L-functions sit at the meeting point of all these theories and have played a profound role in shaping the evolution of number theory. This book presents a big picture of zeta and L-functions and the complex theories surrounding them, combining standard material with results and perspectives that are not made explicit elsewhere in the literature. Particular attention is paid to the development of the ideas surrounding zeta and L-functions, using quotes from original sources and comments throughout the book, pointing the reader towards the relevant history. Based on an advanced course given at Jussieu in 2013, it is an ideal introduction for graduate students and researchers to this fascinating story.

In Search of the Riemann Zeros

The Riemann zeta-function is our most important tool in the study of prime numbers, and yet the famous “Riemann hypothesis” at its core remains unsolved. This book studies the theory from every angle and includes new material on recent work.

Multiple Zeta Functions, Multiple Polylogarithms and Their Special Values

The Riemann Hypothesis

Zeta and L -functions in Number Theory and Combinatorics

This book introduces prime numbers and explains the famous unsolved Riemann hypothesis.

The Zeta Function Of Riemann

Zeta and Q-Zeta Functions and Associated Series and Integrals

In this text, the famous zeros of the Riemann zeta function and its generalizations (L-functions, Dedekind and Selberg zeta functions) are analyzed through several zeta functions built over...
Lectures on the Riemann Zeta Function

In August 1859 Bernhard Riemann, a little-known 32-year old mathematician, presented a paper to the Berlin Academy titled: "On the Number of Prime Numbers Less Than a Given Quantity." In the middle of that paper, Riemann made an incidental remark—a guess, a hypothesis. What he tossed out to the assembled mathematicians that day has proven to be almost cruelly compelling to countless scholars in the ensuing years. Today, after 150 years of careful research and exhaustive study, the question remains. Is the hypothesis true or false? Riemann's basic inquiry, the primary topic of his paper, concerned a straightforward but nevertheless important matter of arithmetic—a precise formula to track and identify the occurrence of prime numbers. But it is that incidental remark—the Riemann Hypothesis—that is the truly astonishing legacy of his 1859 paper. Because Riemann was able to see beyond the pattern of the primes to discern traces of something mysterious and mathematically elegant shrouded in the shadows—subtle variations in the distribution of those prime numbers. Brilliant for its clarity, astounding for its potential consequences, the Hypothesis took on enormous importance in mathematics. Indeed, the successful solution to this puzzle would herald a revolution in prime number theory. Proving or disproving it became the greatest challenge of the age. It has become clear that the Riemann Hypothesis, whose resolution seems to hang tantalizingly just beyond our grasp, holds the key to a variety of scientific and mathematical investigations. The making and breaking of modern codes, which depend on the properties of the prime numbers, have roots in the Hypothesis. In a series of extraordinary developments during the 1970s, it emerged that even the physics of the atomic nucleus is connected in ways not yet fully understood to this strange conundrum. Hunting down the solution to the Riemann Hypothesis has become an obsession for many—the veritable "great white whale" of mathematical research. Yet despite determined efforts by generations of mathematicians, the Riemann Hypothesis defies resolution. Alternating passages of extraordinarily lucid mathematical exposition with chapters of elegantly composed biography and history, Prime Obsession is a fascinating and fluent account of an epic mathematical mystery that continues to challenge and excite the world. Posited a century and a half ago, the Riemann Hypothesis is an intellectual feast for the cognoscenti and the curious alike. Not just a story of numbers and calculations, Prime Obsession is the engrossing tale of a relentless hunt for an elusive proof—a* and those who have been consumed by it.

The Theory of the Riemann Zeta-function

Exploring the Riemann Zeta Function: 190 years from Riemann's Birth presents a collection of chapters contributed by eminent experts devoted to the Riemann Zeta Function, its generalizations, and their various applications to several scientific disciplines, including Analytic Number Theory, Harmonic Analysis, Complex Analysis, Probability Theory, and related subjects. The book focuses on both old and new results towards the solution of long-standing problems as well as it features some key historical remarks. The purpose of this volume is to present in a unified way broad and deep areas of research in a self-contained manner. It will be particularly useful for graduate courses and seminars as well as it will make an excellent reference tool for graduate students and researchers in Mathematics, Mathematical Physics, Engineering and Cryptography.

Exploring the Riemann Zeta Function

There are still many arithmetic mysteries surrounding the values of the Riemann zeta function at the odd positive integers greater than one. For example, the matter of their irrationality, let alone transcendence, remains largely unknown. However, by extending ideas of Garland, Borel proved that these values are related to the higher K-theory of the ring of integers. Shortly afterwards, Bloch and Kato proposed a Tamagawa number-type conjecture for these values, and showed that it would follow from a result in motivic cohomology which was unknown at the time. This vital result from motivic cohomology was subsequently proven by Huber, Kings, and Wildeshaus. Bringing together key results from K-theory, motivic cohomology, and Iwasawa theory, this book is the first to give a complete proof, accessible to graduate students, of the Bloch–Kato conjecture for odd positive integers. It includes a new account of the results from motivic cohomology by Huber and Kings.

Probability Theory and Mathematical Statistics

This work has been selected by scholars as being culturally important and is part of the knowledge base of civilization as we know it. This work is in the public domain in the United States of America, and possibly other nations. Within the United States, you may freely copy and distribute this work, as no entity (individual or corporate) has a copyright on the body of the work. Scholars believe, and we concur, that this work is important enough to be preserved, reproduced, and made generally available to the public. To ensure a quality reading experience, this work has been proofread and republished using a format that seamlessly blends the original graphical elements with text in an easy-to-read typeface. We appreciate your support of the preservation process, and thank you for being an important part of keeping this knowledge alive and relevant.
The Theory of the Riemann Zeta-function

Number theory is one of the largest and most popular subject areas in mathematics, and this book is a superb entry to the subject. It features a well-known international author and covers enough material to satisfy both students and the serious researcher. A splendid addition to the marque series of the AMS publishing program.

Number Theory

Superb high-level study of one of the most influential classics in mathematics examines landmark 1859 publication entitled "On the Number of Primes Less Than a Given Magnitude," and traces developments in theory inspired by it. Topics include Riemann's main formula, the prime number theorem, the Riemann-Siegel formula, large-scale computations, Fourier analysis, and other related topics. English translation of Riemann's original document appears in the Appendix.

Prime Obsession

The Riemann Hypothesis has become the Holy Grail of mathematics in the century and a half since 1859 when Bernhard Riemann, one of the extraordinary mathematical talents of the 19th century, originally posed the problem. While the problem is notoriously difficult, and complicated even to state carefully, it can be loosely formulated as "the number of integers with an even number of prime factors is the same as the number of integers with an odd number of prime factors." The Hypothesis makes a very precise connection between two seemingly unrelated mathematical objects, namely prime numbers and the zeros of analytic functions. If solved, it would give us profound insight into number theory and, in particular, the nature of prime numbers. This book is an introduction to the theory surrounding the Riemann Hypothesis. Part I serves as a compendium of known results and as a primer for the material presented in the 20 original papers contained in Part II. The original papers place the material into historical context and illustrate the motivations for research on and around the Riemann Hypothesis. Several of these papers focus on computation of the zeta function, while others give proofs of the Prime Number Theorem, since the Prime Number Theorem is so closely connected to the Riemann Hypothesis. The text is suitable for a graduate course or seminar or simply as a reference for anyone interested in this extraordinary conjecture.

Random Matrices, Frobenius Eigenvalues, and Monodromy

Zeta and L-functions play a central role in number theory. They provide important information of arithmetic nature. This book, which grew out of the author's teaching over several years, explores the interaction between number theory and combinatorics using zeta and L-functions as a central theme. It provides a systematic and comprehensive account of these functions in a combinatorial setting and establishes, among other things, the combinatorial counterparts of celebrated results in number theory, such as the prime number theorem and the Chebotarev density theorem. The spectral theory for finite graphs and higher dimensional complexes is studied. Of special interest in theory and applications are the spectrally extremal objects, called Ramanujan graphs and Ramanujan complexes, which can be characterized by their associated zeta functions satisfying the Riemann Hypothesis. Explicit constructions of these extremal combinatorial objects, using number-theoretic and combinatorial means, are presented. Research on zeta and L-functions for complexes other than graphs emerged only in recent years. This is the first book for graduate students and researchers offering deep insight into this fascinating and fast developing area.

Spectral Theory of the Riemann Zeta-Function

Formulated in 1859, the Riemann Hypothesis is the most celebrated and multifaceted open problem in mathematics. In essence, it states that the primes are distributed as harmoniously as possible—or, equivalently, that the Riemann zeros are located on a single vertical line, called the critical line. In this book, the author proposes a new approach to understand and possibly solve the Riemann Hypothesis. His reformulation builds upon earlier (joint) work on complex fractal dimensions and the vibrations of fractal strings, combined with string theory and noncommutative geometry. Accordingly, it relies on the new notion of a fractal membrane or quantized fractal string, along with the modular flow on the associated moduli space of fractal membranes. Conjecturally, under the action of the modular flow, the spacetime geometries become increasingly symmetric and crystal-like, hence, arithmetic. Correspondingly, the zeros of the associated zeta functions eventually condense onto the critical line, towards which they are attracted, thereby explaining why the Riemann Hypothesis must be true. Written with a diverse audience in mind, this unique book is suitable for graduate students, experts and nonexperts alike, with an interest in number theory, analysis, dynamical systems, arithmetic, fractal or noncommutative geometry, and mathematical or theoretical physics.

Zeta Functions of Groups and Rings

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The Riemann Zeta Function is one of the most studied objects in mathematics, and is of fundamental importance. In this book, based on his own research, Professor Motohashi shows that the function is closely bound with automorphic forms and that many results from there can be woven with techniques and ideas from analytic number theory to yield new insights into, and views of, the zeta function itself. The story starts with an elementary but unabridged treatment of the spectral resolution of the non-Euclidean Laplacian and the trace formulas. This is achieved by the use of standard tools from analysis rather than any heavy machinery, forging a substantial aid for beginners in spectral theory as well. These ideas are then utilized to unveil an image of the zeta-function, first perceived by the author, revealing it to be the main gem of a necklace composed of all automorphic L-functions. In this book, readers will find a detailed account of one of the most fascinating stories in the development of number theory, namely the fusion of two main fields in mathematics that were previously studied separately.

Zeta Functions over Zeros of Zeta Functions

This book provides both classical and new results in Reimann Zeta-Function theory, one of the most important problems in analytic number theory. These results have application in solving problems in multiplicative number theory, such as power moments, the zero-free region, and the zero density estimates. The book also furnishes annotated proofs, end-of-chapter notes, historical discussions and references.

Quantized Number Theory, Fractal Strings And The Riemann Hypothesis: From Spectral Operators To Phase Transitions And Universality

The study of the subgroup growth of infinite groups is an area of mathematical research that has grown rapidly since its inception at the Groups St. Andrews conference in 1985. It has become a rich theory requiring tools from and having applications to many areas of group theory. Indeed, much of this progress is chronicled by Lubotzky and Segal within their book [42]. However, one area within this study has grown explosively in the last few years. This is the study of the zeta functions of groups with polynomial subgroup growth, in particular, torsion-free, finitely-generated nilpotent groups. These zeta functions were introduced in [32], and other key papers in the development of this subject include [10, 17], with [19, 23, 15] as well as [42] presenting surveys of the area. The purpose of this book is to bring into print significant and as yet unpublished work from three areas of the theory of zeta functions of groups. First, there are now numerous calculations of zeta functions of groups by doctoral students of the first author, which are yet to be made into printed form outside their theses. These explicit calculations provide evidence in favour of conjectures, or indeed can form inspiration and evidence for new conjectures. We record these zeta functions in Chap. 2. In particular, we document the functional equations frequently satisfied by the local factors. Explaining this phenomenon is, according to the first author and Segal [23], “one of the most intriguing open problems in the area”.

The Theory of the Riemann Zeta-function

All needed notions are developed within the book: with the exception of fundamentals which are presented in introductory lectures, no other knowledge is assumed. Provides a more in-depth introduction to the subject than other existing books in this area. Over 400 exercises including hints for solutions are included.

Spectral Theory of the Riemann Zeta-Function

The aim of the series is to present new and important developments in pure and applied mathematics. Well established in the community over two decades, it offers a large library of mathematics including several important classics. The volumes supply thorough and detailed expositions of the methods and ideas essential to the topics in question. In addition, they
Contributions to the Theory of the Riemann Zeta-function

This text covers exponential integrals and sums, 4th power moment, zero-free region, mean value estimates over short intervals, higher power moments, omega results, zeros on the critical line, zero-density estimates, and more. 1985 edition.

Limit Theorems for the Riemann Zeta-Function

An introduction to the analytic techniques used in the investigation of zeta functions through the example of the Riemann zeta function. It emphasizes central ideas of broad application, avoiding technical results and the customary function-theoretic approach.

Zeta Functions of Graphs

This is the first introductory book on multiple zeta functions and multiple polylogarithms which are the generalizations of the Riemann zeta function and the classical polylogarithms, respectively, to the multiple variable setting. It contains all the basic concepts and the important properties of these functions and their special values. This book is aimed at graduate students, mathematicians and physicists who are interested in this current active area of research. The book will provide a detailed and comprehensive introduction to these objects, their fascinating properties and interesting relations to other mathematical subjects, and various generalizations such as their q-analogs and their finite versions (by taking partial sums modulo suitable prime powers). Historical notes and exercises are provided at the end of each chapter. Contents: Multiple Zeta Functions, Multiple Polylogarithms (MPLs), Multiple Zeta Values (MZVs), Drinfeld Associator and Single-Valued MZVs, Multiple Zeta Value Identities, Symmetrized Multiple Zeta Values (SMZVs), Multiple Harmonic Sums (MHSs) and Alternating Version, Finite Multiple Zeta Values and Finite Euler Sums, Analogos of Multiple Harmonic (Star) Sums. The book provides for the first time a comprehensive introduction to multiple polylogarithms and their special values at roots of unity, from the basic definitions to the more advanced topics in current active research. The book contains a few quite intriguing results relating the special values of multiple zeta functions and multiple polylogarithms to other branches of mathematics and physics, such as knot theory and the theory of motives. Many exercises contain supplementary materials which deepen the reader's understanding of the main text.

The Theory of the Riemann Zeta-function

Zeta and q-Zeta Functions and Associated Series and Integrals is a thoroughly revised, enlarged and updated version of Series Associated with the Zeta and Related Functions. Many of the chapters and sections of the book have been significantly modified or rewritten, and a new chapter on the theory and applications of the basic (or q-) extensions of various special functions is included. This book will be invaluable because it covers not only detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions, but stimulating historical accounts of a large number of problems and well-classified tables of series and integrals. Detailed and systematic presentations of the theory and applications of the various methods and techniques used in dealing with many different classes of series and integrals associated with the Zeta and related functions.

Some Aspects of the Theory of the Riemann Zeta Function

The Distribution of Prime Numbers